Roll No. ☐☐☐☐☐☐☐☐☐☐☐☐ **Total No. of Pages : 03**

Total No. of Questions : 07

M.Sc. (Applied Mathematics) (Sem.–4)

DISCRETE MATHEMATICS

Subject Code : MSM-511

M.Code : 75979

Time : 3 Hrs. **Max. Marks : 80**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of EIGHT questions carrying TWO marks each.**
2. **SECTION - B & C have THREE questions in each section carrying SIXTEEN marks each.**
3. **Select atleast TWO questions from SECTION - B & C EACH.**

SECTION-A

Q1. Write short notes on :

(a) Define the Fibonacci sequence and find its generating function.

(b) Which sequence has the generating function $\frac{1}{1-z-z^2}$?

(c) Reduce the expression into simpler form by using the rules of Boolean algebra $AB + A\overline{C} + \overline{A}\,\overline{B}C(AB + C)$.

(d) Prove that the complement of every element on a Boolean algebra B is unique.

(e) Prove that the maximum degree of edges in a graph G with n vertices and no multiple edges are $\frac{n(n-1)}{2}$.

(f) Prove that the maximum number of nodes in a binary tree of depth d is $2^d - 1$, where d [illegible] 1.

(g) How many edges must be drawn in order to obtain a planar graph with 5 vertices that define 7 regions? Draw such graph.

(h) What is an internal vertex, a leaf and a subtree in a rooted tree?

SECTION-B

Q2. (a) Find the disjunctive normal form of $(p \wedge q) \downarrow (\vee p \quad r)$. Using inference theory prove that r is a valid conclusion following from the hypothesis $(p \wedge q) \quad r,\ p \quad s, q \vee s, \quad s$.

(b) By finding generating function of the sequence {S(n)}. find the solution of the recurrence relation S(n + 2) - 7S(n + 1) + 12 S(n) = 0, where S(0) = 2, S (1) = 5.

Q3. (a) Consider the lattice D_{60} = {1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60}

a) Draw the Hasse Diagram of D_{60}.

b) Find all Join-irreducible elements.

c) Find all atoms.

d) Find complements of 2 and 10, if they exist.

e) Express each x as a join of irredundant join-irreducible elements.

(b) Consider the Boolean expression $f(x,y,z) = z(x' + y) + y'$. Reduce it into sum-of-product form and hence to complete sum-of-product form.

Q4. (a) An undirected graph possesses an Eulerian circuit if and only if it is connected and its vertices are all of even degree. Prove with the help of a suitable example.

(b) Show that $e: B^3 \downarrow B^4$ defined as e(000) = 1100, e(100) = 1001.

e(010) = 1010, e(001) = 0011, e(110) = 1111, e(101) = 0110, e(011) = 0101, e(111) = 0000 is a group code. Find how many errors e can detect.

SECTION-C

Q5. (a) Solve S(K) – 4S(K-1) + 4S(K-2) = 3K + 2^K, where S(0) = 1, S(1) = 1.

(b) Find the circuit diagram of $f(x,y,z) = xy' + x'z$.

Q6. (a) Consider a Boolean expression $f(a,b,c) = ((ab)'c)'((a'+c)(b'+c'))'$.

Reduce it into a sum-of-product form.

(b) State and prove Euler's Formula for graphs.

Q7. (a) Show that the graph K_5 is non-planar.

(b) Let $e: B^m \downarrow B^n$ be a group code. Then show that the minimum distance of e is the minimum weight of a non zero code word.

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.